Ruby Language Simulation of Fourier's Law of Conduction

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Abstract

Fourier's Law of Conduction is simulated using the Ruby language and seperately by a Ruby on Rails web application. Results for several sets of parameters are generated, demonstrating the effect of various factors on heat flow.

1 History and The Law

Fourier's Law of Conduction states how much heat shall flow through a conductor bridging a thermal difference. This is a simple, well-known law found in most introductory physics textbooks.

1.1 The Law

Heat flow involves the transfer of randomized, molecular-level kinetic energy. Fourier's Law states that heat flow is proportional to the conductor's area and the thermal difference bridged, and is inversely proportional to the conductor's length.[1] This law was formulated by Joseph Fourier in 1822, [2] and can be stated as:

$$\frac{dQ}{dt} = k\Delta T\frac{A}{l} \tag{1}$$

where Q is heat energy, t is time, k is a constant of proportionality dependent upon the conductor material, T is temperature difference, A is area and l is length.

1.2 Significance of the Law

Fourier's Law is important because it describes heat flow as a function of temperature difference, or more particularly of temperature gradient $\bigtriangledown T$. It does not matter how high or low the temperatures involved are, but only their difference. Also, the relationship between conductor dimensions and heat flow are shown to be simple. Doubling the length of the conductor halves the heat flow. Doubling the area doubles the flow.

2 The simulations

The simulations take as paramters the temperature of each of two inexhaustible thermal reservoirs, and the conductor's material, length and area. The simulation looks up the coefficient of conduction for the material.

2.1 Ruby simulation

A version of the simulation written in Ruby is suitable for students who know how to code or are able and willing to learn. Ruby is a good initial language to learn, because it is simple to use, understand and read. Yet it contains many of the features of modern, higher level languages such as object-oriented programming.

The Ruby language simulation is written in version 1.9.2. It has been placed on Github in a publicly available, open-source repository. The code can be downloaded and run on the user's machine. A command line or terminal utility is required, and Ruby 1.9.2 or higher will need to be installed if it is not already present. The Ruby code can be altered or branched as desired. The Ruby simulation code can be found at:

https://github.com/mciotola/fouriers_law_of_heat_conduction

2.2 Ruby on Rails simulation

Another version of the simulation has been written in Ruby on Rails version 3.1 (using Ruby version 1.9.2) and placed on the internet as a web application. It can be used by anyone with a compatible browser. It is especially suitable for students who do not know how to use spreadsheets or run programs, but is useful for anyone who wants to quickly use Fourier's Law. The Ruby on Rails simulation can be found at:

http://www.heatsuite.com/?page_id=64

3 Sample results

Default paramaters for this simulation are for a warmer reservoir temperature of 500 K, a cooler reservoir temperature of 300 K, and a copper conductor of 0.05 meters long and 0.001 m^2 (1 cm^2). The parameters can be changed as desired. For the default values, the thermal conductivity of copper is 160.0 W/m/K. This simulation produces a single number representing the heat flow across the conductor. For the above set of paramaters, a result of 160 J/s is produced.

An advantage of this simulator is that it allows the user to rapidly try out different parameters. So tables of various combinations can be built quickly. This allows the user to better understand the relationships between particular parameters and heat flow. For example, the temperature of the warmer reservoir can be increased, while keeping the other default values. Using the simulator, this can be accomplished without using a spreadsheet or direct programming; only a compatible web browser is required. Table 1 shows results for different temperatures.

warmer temperature (K)	cooler temperature (K)	heat flow (J/s)
300	300	0
300	400	80
300	500	160
300	600	240
3000	3200	160
30000	30200	160
300000	300200	160
300	500	160
3000	5000	1,600
30000	30000	16,000

Table 1: Heat Flow for various reservoir temperatures for a copper conductor

By trying different ranges of temperatures, the pattern becomes apparent: heat flow is only a function of the difference of temperatures. The magnitude of temperatures does not matter.

Table 2 shows the effect of conductor area upon heat flow. Doubling the area doubles heat flow. Increasing area by an order of magnitude likewise increases heat flow by an order of magnitude. The temperature difference is held constant to better illustrate the effect on area. Using the default values for temperatures, area and material make it easier to run the sumulation for numerous areas.

warmer temperature (K)	cooler temperature (K)	area (m^2)	heat flow (J/s)
300	500	0.0000	0
300	500	0.0001	160
300	500	0.0002	320
300	500	0.0003	480
300	500	0.0004	640
300	500	0.0001	160
300	500	0.001	1,600
300	500	0.01	16,000
300	500	0.1	160,000
300	500	1	1,600,000

Table 2: Heat Flow for various copper conductor areas

Table 3 shows the effect of conductor length upon heat flow. This may be a somewhat more challenging concept for those new to thermodynamics, because division rather than simple multiplication is involved. Yet the user is able to see that doubling length halves heat flow. The simulation is able to meaningfully handle division by zero.

Table 4 shows the effect of conductor material upon heat flow. The simulation only contains a choice of three materials. However, additional materials and their thermal conductivities can easily be added to the Ruby code.

warmer temperature (K)	cooler temperature (K)	length (m)	heat flow (J/s)
300	500	0.00	infinity
300	500	0.05	160
300	500	0.10	80
300	500	0.15	53
300	500	0.20	40

Table 3: Heat Flow for various copper conductor lengths

warmer temperature (K)	cooler temperature (K)	length (m)	heat flow (J/s)
300	500	copper	160
300	500	iron	32
300	500	wood	0.032

Table 4: Heat Flow for various conductor materials

4 Conclusions and Uses

These are extremely simple simulations, yet they demonstrates both a core thermodynamic principle and basic computer simulations. Hence this simulation can be used to help introduce both Fourier's Law of Conduction and programming.

References

[1] Schroeder, Daniel V., *An Introduction to Thermal Physics*, 2000.

[2] Davies, T.W., *Fourier's Law*, Thermopedia.com, http://www.thermopedia.com/content/781/ (last viewed on February 18, 2014).